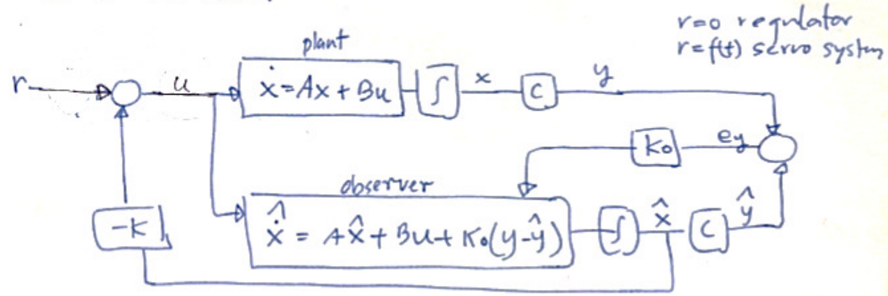
EN5101 Digital Control Systems
Observer Design

Prof. Rohan Munasinghe
Dept of Electronic and Telecommunication Engineering
University of Moratuwa

State Observer

- Sometimes not all states are available for measurement
- May be it is not practical to measure (or too expensive) all states.
- Observer (or estimator) is another dynamical system made of RLC components to produce estimated state variables.



Observer, Plant, and Control Law

Observer : adjust estimated output $\hat{y}$ by looking at output error $e_y = y - \hat{y}$, in order to quickly reduce it.

$$\dot{\hat{x}} = A\hat{x} + Bu + K_o(y-\hat{y}) \;;\; \hat{x}(0) = \hat{x}_0 \quad -①$$
$$\hat{y} = C\hat{x}$$

Plant

$$\dot{x} = Ax + Bu \quad -②$$
$$y = Cx$$

Control law

$$u = r - K\hat{x}$$ (actual states are not available for measurement)

② - ①

$$\dot{x} - \dot{\hat{x}} = A(x-\hat{x}) - K_o(y-\hat{y})$$
$$= A(x-\hat{x}) - K_oC(x-\hat{x})$$
$$\dot{x} - \dot{\hat{x}} = (A - K_oC)(x-\hat{x})$$
$$\dot{e} = (A - K_oC)e \quad \text{Error dynamics}$$

Set $K_o = \begin{bmatrix} K_{o1} \\ \vdots \\ K_{on} \end{bmatrix}$ so that $e \to 0$ as quickly as possible

$|sI - A + K_oC| = 0$ poles ?

If observer poles are faster than the system poles, accurate state estimate is reached quickly

System (A,C) should be completely observable in order to be able to determine an the observer gain matrix K_o to arbitrarily locate observer poles.

lets say the desired observer poles are known as $\mu_1, \mu_2 \cdots \mu_n$, Then $(s+\mu_1)(s+\mu_2)\cdots(s+\mu_n) = |sI - A + K_oC|$ yields the elements of matrix K_o

Example : the system

$$\dot{x} = Ax + Bu \qquad A = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix}$$
$$y = Cx$$
$$u = -K\hat{x} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

lets design a full state feedback with an observer, whose poles are located at -10. $\quad C = [0 \;\; 1]$

check for system observability $O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ rank 2

∴ completely observable

It is possible to design a full-state feedback

cha^c eq^n $|sI - A + K_oC| = (s+10)^2 \leftarrow$ desired cha^c eq^n

$$\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 20.6 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} k_{01} \\ k_{02} \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \right| = s^2 + 20s + 100$$

$$\begin{vmatrix} s & -20.6 + k_{01} \\ -1 & s + k_{02} \end{vmatrix} = s^2 + 20s + 100$$

$$s^2 + k_{02}s - 20.6 + k_{01} = s^2 + 20s + 100$$

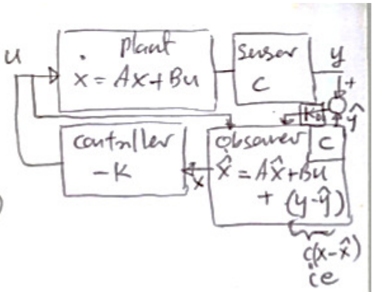
∴ $k_{02} = 20$, $k_{01} - 20.6 = 100 \Rightarrow k_{01} = 120.6$

The observer gain matrix is

$$K_o = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

Plant with full order observer :

$$\dot{x} = Ax - BK\hat{x}$$
$$= Ax - BK(x - e) \quad \text{as } x - \hat{x} = e$$
$$= (A - BK)x + BKe \quad \text{—— (1)}$$



Full order observer :

$$\dot{e} = (A - K_oC)e \quad \text{—— (2) from error dynamics}$$

(1) and (2) can be arranged as follows

$$\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \overbrace{\begin{pmatrix} A - BK & BK \\ 0 & A - K_oC \end{pmatrix}}^{A^*} \begin{pmatrix} x \\ e \end{pmatrix}$$

Characteristic eq^n $\det[sI - A^*] = 0$

$$\det\left[\begin{pmatrix} sI & 0 \\ 0 & sI \end{pmatrix} - \begin{pmatrix} A - BK & BK \\ 0 & A - K_oC \end{pmatrix}\right] = 0$$

$$\det\begin{bmatrix} sI - A + BK & -BK \\ 0 & sI - A + K_oC \end{bmatrix} = 0$$

Block triangular matrix determinant can be written as follows

$$\underbrace{\det(sI - A + BK)}_{\text{cha}^c \text{ eq}^n \text{ of the plant}} . \underbrace{\det(sI - A + K_oC)}_{\text{cha}^c \text{ eq}^n \text{ of the observer}} = 0$$

Compensated System has poles of the plant and estimator. Estimator poles can be independently located.

Observer Based Controller

- what affects the feedback of observer estimates, than feeding back actual state?

system: $\dot{x} = Ax + Bu$ $\quad u = -K\hat{x}$ from the observer — ①

$y = Cx$

observer: $\dot{\hat{x}} = A\hat{x} + Bu + K_o(y - \hat{y})$

$= A\hat{x} + B(-K\hat{x}) + K_o(y - C\hat{x})$

$\dot{\hat{x}} = (A - BK - K_oC)\hat{x} + K_o y$

Laplace, with initial state zero (at the observer)

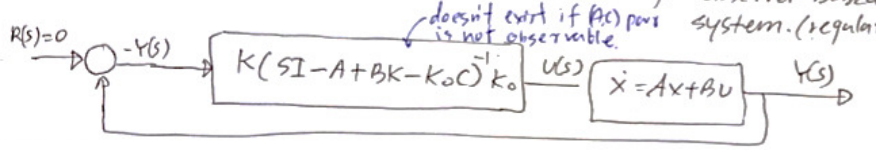
$s\hat{X}(s) - \overset{0}{\hat{x}(0)} = (A - BK - K_oC)\hat{X}(s) + K_o Y(s)$

$\hat{X}(s) = (sI - A + BK + K_oC)^{-1} K_o Y(s)$ — ②

put ② on $u = -K\hat{x}$ in ① in Laplace domain

$U(s) = -K(sI - A + BK + K_oC)^{-1} K_o Y(s)$

$\dfrac{U(s)}{-Y(s)} = K(sI - A + BK + K_oC)^{-1} K_o$ is the controller for observer based system. (regulator)



Assignment #2

: Design a feedback (full-state) for the system

$\dot{x} = Ax + Bu$ $\quad A = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad C = [1 \;\; 0]$

$y = Cx$

(1) to locate closed-loop poles at $1.8 \pm j2.4$ [assume all states are available for measurement]

(2) Design an observer with its closed loop poles located at -8

(3) Determine the observer-~~controller~~ based controller ~~regulator~~.

Notes

1. observer-controller design is not unique solution method, rather several sets of closed-loop poles should be considered and selected the one that gives best performance.
2. Observer-controller has wider system bandwidth that allows higher frequency inclusion, noise etc, due to fast poles in the observer.
3. Observer generally reduces relative stability which might be a problem.
4. observer-controller is always of higher-order compared to compensators of low order that are resulted from classical methods, such as root-locus. Therefore, low order compensators should be first considered, and if it fails to perform well, then consider an observer. (higher order compensators are expensive)
5. Fast poles located by the observer-controller might lead to controller saturation.